

数値計算・講義資料 —代数方程式に対するDK法—

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代数方程式

$$z^5 - 10z^4 + 35z^3 - 50z^2 + 24z = z(z-1)(z-2)(z-3)(z-4) = 0 \quad (1)$$

の根を2次DK法で求めた(倍精度計算). 初期解は次の通り:

$$z_j^{(0)} = 2 + 7 \exp\left(\frac{i}{5} \left(2\pi(j-1) + \frac{3}{2}\right)\right), \quad j = 1, \dots, 5. \quad (2)$$

反復解とその誤差は次の通り:

nu = 0

z(1) = (8.687355423879243e+000, 2.068641446629377e+000) error = 5.124e+000

z(2) = (2.099111545709064e+000, 6.999298314938945e+000) error = 7.000e+000

z(3) = (-4.626101119953502e+000, 2.257162809402759e+000) error = 5.147e+000

z(4) = (-2.194267250734073e+000, -5.604294980585840e+000) error = 6.451e+000

z(5) = (6.033901401099266e+000, -5.720807590385240e+000) error = 6.476e+000

nu = 1

z(1) = (7.484911161239074e+000, 1.614522986461800e+000) error = 3.841e+000

z(2) = (2.081410964958984e+000, 5.454265571578484e+000) error = 5.455e+000

z(3) = (-3.434817886474454e+000, 1.761609134882836e+000) error = 3.860e+000

z(4) = (-1.443196691898704e+000, -4.369876902960954e+000) error = 5.006e+000

z(5) = (5.311692452175100e+000, -4.460520789962167e+000) error = 5.024e+000

nu = 2

z(1) = (6.477493825540678e+000, 1.263096052999576e+000) error = 2.781e+000

z(2) = (2.066384775191656e+000, 4.271943379411148e+000) error = 4.272e+000

z(3) = (-2.436511607638345e+000, 1.378195690533920e+000) error = 2.799e+000

z(4) = (-8.089071156772804e-001, -3.421077390725535e+000) error = 3.870e+000

z(5) = (4.701540122583292e+000, -3.492157732219111e+000) error = 3.885e+000

nu = 3

z(1) = (5.677914995302257e+000, 9.794713105817958e-001) error = 1.943e+000

z(2) = (2.054412709507618e+000, 3.320894839046229e+000) error = 3.321e+000

z(3) = (-1.644108616212336e+000, 1.068773127131872e+000) error = 1.961e+000

z(4) = (1.330012764973170e-011, 6.947697387504466e-010) error = 2.960e+000

z(5) = (2.609703362725204e-012, 6.115672276219036e-010) error = 2.972e+000

nu = 4

z(1) = (5.059648683935605e+000, 7.450166785727801e-001) error = 1.295e+000

z(2) = (2.045069601847732e+000, 2.540370519465634e+000) error = 2.541e+000

z(3) = (-1.031289203994000e+000, 8.130194252832933e-001) error = 1.313e+000
z(4) = (8.823965900833880e-002,-2.027882117616922e+000) error = 2.223e+000
z(5) = (3.838331259202324e+000,-2.070524505704785e+000) error = 2.234e+000

nu = 5

z(1) = (4.595312318940159e+000, 5.462760269222420e-001) error = 8.080e-001
z(2) = (2.037879374577265e+000, 1.889835356292051e+000) error = 1.890e+000
z(3) = (-5.708504040229547e-001, 5.962536815957125e-001) error = 8.255e-001
z(4) = (3.879855808394718e-001,-1.500058327801984e+000) error = 1.620e+000
z(5) = (3.549673129666059e+000,-1.532306737008021e+000) error = 1.628e+000

nu = 6

z(1) = (4.264192884817923e+000, 3.718026648529043e-001) error = 4.561e-001
z(2) = (2.032414678699425e+000, 1.338342000080318e+000) error = 1.339e+000
z(3) = (-2.422171137854009e-001, 4.058893593081989e-001) error = 4.727e-001
z(4) = (6.124965791982371e-001,-1.046025714070482e+000) error = 1.115e+000
z(5) = (3.333112971069815e+000,-1.070008310170939e+000) error = 1.121e+000

nu = 7

z(1) = (4.054797279513012e+000, 2.123876731360123e-001) error = 2.193e-001
z(2) = (2.028262173491674e+000, 8.618644681324670e-001) error = 8.623e-001
z(3) = (-3.437669805795227e-002, 2.312427774603725e-001) error = 2.338e-001
z(4) = (7.797244456329018e-001,-6.436750272637524e-001) error = 6.803e-001
z(5) = (3.171592799420358e+000,-6.618198914650961e-001) error = 6.837e-001

nu = 8

z(1) = (0.000000000000000e+000, 0.000000000000000e+000) error = 7.801e-002
z(2) = (2.024525855842025e+000, 4.451657273659091e-001) error = 4.458e-001
z(3) = (4.808440098542666e-002, 7.366102499106944e-002) error = 8.797e-002
z(4) = (9.138617849278965e-001,-2.874393424584220e-001) error = 3.001e-001
z(5) = (3.045317270155064e+000,-3.026283654469794e-001) error = 3.060e-001

nu = 9

z(1) = (3.987239674298197e+000,-2.445139599184520e-004) error = 1.276e-002
z(2) = (2.016142484012805e+000, 1.107905993640557e-001) error = 1.120e-001
z(3) = (1.454930070282673e-002,-5.215868401267018e-003) error = 1.546e-002
z(4) = (1.015413461263045e+000,-5.063098664362709e-002) error = 5.293e-002
z(5) = (2.966655079723105e+000,-5.469923035924620e-002) error = 6.406e-002

nu = 10

z(1) = (3.999757272775640e+000,-2.018583104151421e-004) error = 3.157e-004
z(2) = (1.999956045418813e+000,-5.518674238370075e-003) error = 5.519e-003
z(3) = (1.031934261685775e-004,-2.630950259599257e-004) error = 2.826e-004

z(4) = (1.004282772704823e+000, 1.567754198266692e-003) error = 4.561e-003
z(5) = (2.995900715674550e+000, 4.415873376476774e-003) error = 6.025e-003

nu = 11

z(1) = (3.999998929384708e+000, -3.198106053299652e-008) error = 1.071e-006
z(2) = (2.000016455506918e+000, 4.711689822287611e-005) error = 0.000e+000
z(3) = (3.504882544329188e-007, -7.205107351597917e-007) error = 0.000e+000
z(4) = (9.999952579069715e-001, -1.698263675712014e-005) error = 1.763e-005
z(5) = (2.999989006713143e+000, -2.938176967006746e-005) error = 3.137e-005

nu = 12

z(1) = (3.99999999995813e+000, -1.262363067221509e-011) error = 1.330e-011
z(2) = (2.000000000453559e+000, -5.263007444542304e-010) error = 6.948e-010
z(3) = (-2.480989760271727e-012, -8.094698614278468e-013) error = 2.610e-012
z(4) = (1.000000000514083e+000, -3.312599352334471e-010) error = 6.116e-010
z(5) = (2.99999999039027e+000, 8.709937801482440e-010) error = 1.297e-009

nu = 13

z(1) = (4.000000000000001e+000, -5.030401960013204e-021) error = 0.000e+000
z(2) = (2.000000000000004e+000, 1.319379236608951e-018) error = 0.000e+000
z(3) = (-1.289452194244778e-021, 4.226809121563401e-022) error = 0.000e+000
z(4) = (1.000000000000000e+000, -3.986715869412371e-020) error = 0.000e+000
z(5) = (2.99999999999988e+000, -1.274902665650164e-018) error = 0.000e+000

solution

z(1) = (4.000000000000001e+000, -5.030401960013204e-021)
z(2) = (2.000000000000004e+000, 1.319379236608951e-018)
z(3) = (-1.289452194244778e-021, 4.226809121563401e-022)
z(4) = (1.000000000000000e+000, -3.986715869412371e-020)
z(5) = (2.99999999999988e+000, -1.274902665650164e-018)

反復解の収束の様子を図 1(a) に示した。

同じ方程式 (1) の根を今度は 3 次 DK 法で計算した (倍精度計算)。初期解は (2) にとった。反復解は次の通り：

nu = 0

z(1) = (8.687355423879243e+000, 2.068641446629377e+000) error = 5.124e+000
z(2) = (2.099111545709064e+000, 6.999298314938945e+000) error = 7.000e+000
z(3) = (-4.626101119953502e+000, 2.257162809402759e+000) error = 5.147e+000
z(4) = (-2.194267250734073e+000, -5.604294980585840e+000) error = 6.451e+000
z(5) = (6.033901401099266e+000, -5.720807590385240e+000) error = 6.476e+000

nu = 1

z(1) = (6.610016799636648e+000, 1.331918931161915e+000) error = 2.930e+000

z(2) = (2.068319062196092e+000, 4.507608333030068e+000) error = 4.508e+000
z(3) = (-2.567762710721758e+000, 1.453314632616450e+000) error = 2.951e+000
z(4) = (-8.910803476884617e-001,-3.609134130952187e+000) error = 4.075e+000
z(5) = (4.780540970833753e+000,-3.684184030988261e+000) error = 4.092e+000

nu = 2

z(1) = (5.253116992482615e+000, 8.296371629906973e-001) error = 1.503e+000
z(2) = (2.047975936699823e+000, 2.824453889208359e+000) error = 2.825e+000
z(3) = (-1.223013819005062e+000, 9.053491956502305e-001) error = 1.522e+000
z(4) = (-3.403672530351987e-002,-2.256245033103611e+000) error = 2.482e+000
z(5) = (3.955984127311315e+000,-2.303569076649072e+000) error = 2.494e+000

nu = 3

z(1) = (4.426029044041202e+000, 4.661699015662119e-001) error = 6.315e-001
z(2) = (2.035111955783324e+000, 1.636273436351116e+000) error = 1.637e+000
z(3) = (-4.028586892717194e-001, 5.088884642082727e-001) error = 6.490e-001
z(4) = (5.013263273023052e-001,-1.291112054642511e+000) error = 1.384e+000
z(5) = (3.440341528132662e+000,-1.319516372634886e+000) error = 1.391e+000

nu = 4

z(1) = (4.017046394216654e+000, 1.731336787028505e-001) error = 1.740e-001
z(2) = (2.027207286349630e+000, 7.479652417416508e-001) error = 7.485e-001
z(3) = (2.765344429769390e-003, 1.879381280073482e-001) error = 1.880e-001
z(4) = (8.196469742970959e-001,-5.409629966595787e-001) error = 5.702e-001
z(5) = (3.132682503971687e+000,-5.583301547201480e-001) error = 5.739e-001

nu = 5

z(1) = (3.987508837301666e+000,-4.844874300044461e-003) error = 1.340e-002
z(2) = (2.016099416953401e+000, 8.291519748984222e-002) error = 8.446e-002
z(3) = (1.367459072295669e-002,-1.037458743036060e-002) error = 1.716e-002
z(4) = (1.045901099763376e+000,-2.239242602120117e-002) error = 5.107e-002
z(5) = (2.939463593602993e+000,-2.751215616842739e-002) error = 6.649e-002

nu = 6

z(1) = (4.000005683955016e+000, 7.020769779670505e-006) error = 9.033e-006
z(2) = (1.999673992049347e+000,-4.703032794968476e-004) error = 5.722e-004
z(3) = (-2.000045997383251e-006, 1.245161219117288e-005) error = 1.261e-005
z(4) = (9.998194879644497e-001,-6.220420890834175e-005) error = 1.909e-004
z(5) = (3.000264077188462e+000,-2.742865218045287e-004) error = 3.807e-004

nu = 7

z(1) = (3.99999999999971e+000,-1.972008575315196e-014) error = 3.757e-014
z(2) = (1.99999999907932e+000,-6.388517763845220e-011) error = 1.121e-010

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z( 3) = (-2.463772441678403e-014,-4.327208587924427e-014) error = 4.979e-014
z( 4) = ( 9.99999999956940e-001, 2.096170258853933e-011) error = 2.140e-011
z( 5) = ( 3.000000000067094e+000,-5.554632486392749e-011) error = 8.710e-011

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nu = 8
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z( 1) = ( 4.000000000000003e+000,-4.102076707149261e-029) error = 0.000e+000
z( 2) = ( 1.999999999999998e+000, 0.000000000000000e+000) error = 0.000e+000
z( 3) = ( 3.155443620884047e-030, 6.310887241768094e-030) error = 0.000e+000
z( 4) = ( 9.999999999999999e-001, 2.261821987449685e-026) error = 0.000e+000
z( 5) = ( 3.000000000000007e+000,-1.098599251046990e-025) error = 0.000e+000

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solution
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z( 1) = ( 4.000000000000003e+000,-4.102076707149261e-029)
z( 2) = ( 1.999999999999998e+000, 0.000000000000000e+000)
z( 3) = ( 3.155443620884047e-030, 6.310887241768094e-030)
z( 4) = ( 9.999999999999999e-001, 2.261821987449685e-026)
z( 5) = ( 3.000000000000007e+000,-1.098599251046990e-025)

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反復解の収束の様子を図1(b)に示した.

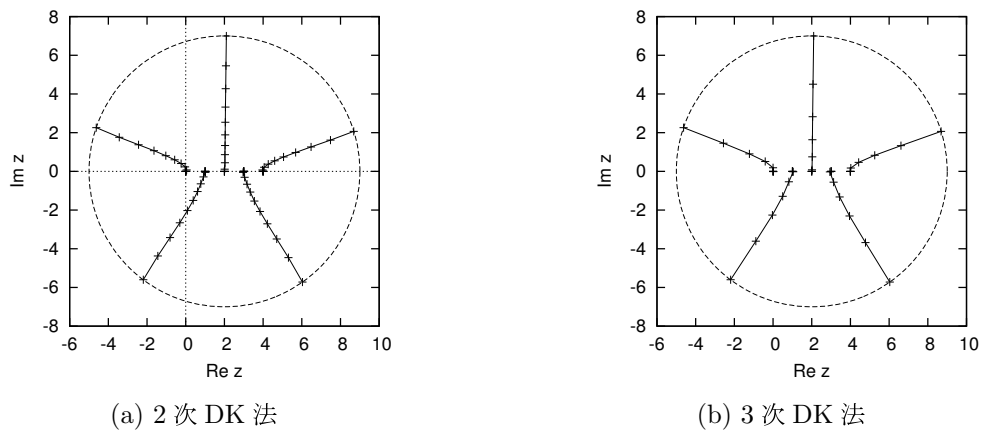


図 1: 代数方程式 (1) に対する DK 法の収束の様子.

同じ方程式 (1) に対し 3 次 DK 法で初期解を

$$z_j^{(0)} = \beta + R \exp\left(i \frac{2\pi(j-1)}{5}\right) \quad (j = 1, \dots, 5)$$

と対称性のある配置にしたもので解いた. 逐次近似解の変化の様子をを??に示す. 対称性のない初期解の場合に比べて解の収束が悪くなっていることがわかる.

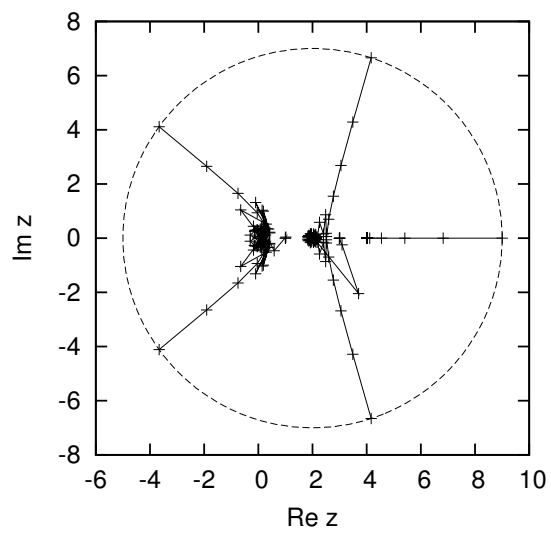


図 2: 代数方程式 (1) に対する DK 法で対称性のある初期値配置に取った場合の, 解の収束の様子.