Ricciテンソル $R_{\mu\nu} = R^{\kappa}_{\ \mu\kappa\nu}$ が対称であること $(R_{\mu\nu} = R_{\nu\mu})$ の証明.

$R_{\kappa\lambda\mu\nu}$ が次のように表されることを用いる（証明は後述）.

$$R_{\kappa\lambda\mu\nu} = \frac{1}{2}(\partial_\kappa\partial_\nu g_{\lambda\mu} + \partial_\lambda\partial_\mu g_{\kappa\nu} - \partial_\kappa\partial_\mu g_{\lambda\nu} - \partial_\lambda\partial_\nu g_{\kappa\mu})$$
$$+ g_{\rho\sigma}(\Gamma^\rho_{\kappa\nu}\Gamma^\sigma_{\lambda\mu} - \Gamma^\rho_{\kappa\mu}\Gamma^\sigma_{\lambda\nu}). \quad \text{——①}$$

これを用いると，$R_{\mu\nu}$ が次のように表されることがわかる.

$$R_{\mu\nu} = \frac{1}{2} g^{\lambda\kappa} R_{\kappa\mu\lambda\nu}$$
$$= \frac{1}{2} g^{\lambda\kappa}(\partial_\kappa\partial_\nu g_{\mu\lambda} + \partial_\mu\partial_\lambda g_{\kappa\nu} - \partial_\kappa\partial_\lambda g_{\mu\nu} - \partial_\mu\partial_\nu g_{\kappa\lambda})$$
$$+ g^{\lambda\kappa} g_{\rho\sigma}(\Gamma^\rho_{\kappa\nu}\Gamma^\sigma_{\mu\lambda} - \Gamma^\rho_{\kappa\lambda}\Gamma^\sigma_{\mu\nu})$$
$$= \frac{1}{2} g^{\kappa\lambda}(\partial_\kappa\partial_\nu g_{\lambda\mu} + \partial_\kappa\partial_\mu g_{\lambda\nu} - \partial_\kappa\partial_\lambda g_{\mu\nu} - \partial_\mu\partial_\nu g_{\kappa\lambda})$$
$$+ g^{\kappa\lambda} g_{\rho\sigma}(\Gamma^\rho_{\kappa\mu}\Gamma^\sigma_{\lambda\nu} - \Gamma^\rho_{\kappa\lambda}\Gamma^\sigma_{\mu\nu}).$$

この式から，明らかに $R_{\mu\nu} = R_{\nu\mu}$ である.

〈① の証明〉

$$R_{\kappa\lambda\mu\nu} = g_{\kappa\alpha}R^\alpha_{\ \lambda\mu\nu} = g_{\kappa\alpha}(\partial_\mu\Gamma^\alpha_{\nu\lambda} - \partial_\nu\Gamma^\alpha_{\mu\lambda} + \Gamma^\alpha_{\mu\rho}\Gamma^\rho_{\nu\lambda} - \Gamma^\alpha_{\nu\rho}\Gamma^\rho_{\mu\lambda}).$$

$$g_{\kappa\alpha}\partial_\mu\Gamma^\alpha_{\nu\lambda} = \frac{1}{2} g_{\kappa\alpha}\partial_\mu\left[g^{\alpha\beta}(\partial_\nu g_{\beta\lambda} + \partial_\lambda g_{\beta\nu} - \partial_\beta g_{\nu\lambda})\right]$$
$$= \frac{1}{2}\underbrace{g_{\kappa\alpha}g^{\alpha\beta}}_{\delta^\beta_\kappa}\partial_\mu(\partial_\nu g_{\beta\lambda} + \partial_\lambda g_{\beta\nu} - \partial_\beta g_{\nu\lambda})$$
$$+ \frac{1}{2} g_{\kappa\alpha}\partial_\mu g^{\alpha\beta}(\partial_\nu g_{\beta\lambda} + \partial_\lambda g_{\beta\nu} - \partial_\beta g_{\nu\lambda})$$
$$= \frac{1}{2}(\partial_\mu\partial_\nu g_{\kappa\lambda} + \partial_\lambda\partial_\mu g_{\kappa\nu} - \partial_\mu\partial_\kappa g_{\nu\lambda})$$
$$+ \frac{1}{2} g_{\kappa\alpha}\partial_\mu g^{\alpha\beta}(\partial_\nu g_{\beta\lambda} + \partial_\lambda g_{\beta\nu} - \partial_\beta g_{\nu\lambda}),$$

$$g_{\kappa\alpha}(\partial_\mu\Gamma^\alpha_{\nu\lambda} - \partial_\nu\Gamma^\alpha_{\mu\lambda})$$
$$= \frac{1}{2}(\cancel{\partial_\mu\partial_\nu g_{\kappa\lambda}} + \partial_\lambda\partial_\mu g_{\kappa\nu} - \partial_\kappa\partial_\mu g_{\nu\lambda}) + \frac{1}{2} g_{\kappa\alpha}\partial_\mu g^{\alpha\beta}(\partial_\nu g_{\beta\lambda} + \partial_\lambda g_{\beta\nu} - \partial_\beta g_{\nu\lambda})$$
$$- \frac{1}{2}(\cancel{\partial_\mu\partial_\nu g_{\kappa\lambda}} + \partial_\lambda\partial_\nu g_{\kappa\mu} - \partial_\kappa\partial_\nu g_{\mu\lambda}) - \frac{1}{2} g_{\kappa\alpha}\partial_\nu g^{\alpha\beta}(\partial_\mu g_{\beta\lambda} + \partial_\lambda g_{\beta\mu} - \partial_\beta g_{\mu\lambda})$$
$$= \frac{1}{2}(\partial_\kappa\partial_\nu g_{\lambda\mu} + \partial_\lambda\partial_\mu g_{\kappa\nu} - \partial_\kappa\partial_\mu g_{\lambda\nu} - \partial_\lambda\partial_\nu g_{\kappa\mu})$$
$$+ \frac{1}{2} g_{\kappa\alpha}\partial_\mu g^{\alpha\beta}(\partial_\nu g_{\beta\lambda} + \partial_\lambda g_{\beta\nu} - \partial_\beta g_{\nu\lambda}) - \frac{1}{2} g_{\kappa\alpha}\partial\, g^{\alpha\beta}(\partial_\mu g_{\beta\lambda} + \partial_\lambda g_{\beta\mu} - \partial_\beta g_{\mu\lambda})$$

により，

$$
\left.
\begin{aligned}
R_{\kappa\lambda\mu\nu} = {} & \frac{1}{2}\left(\partial_\kappa \partial_\nu g_{\lambda\mu} + \partial_\lambda \partial_\mu g_{\kappa\nu} - \partial_\kappa \partial_\mu g_{\lambda\nu} - \partial_\lambda \partial_\nu g_{\kappa\mu}\right) \\
& + \frac{1}{2} g_{\kappa\alpha} \partial_\mu g^{\alpha\beta}\left(\partial_\nu g_{\beta\lambda} + \partial_\lambda g_{\beta\nu} - \partial_\beta g_{\nu\lambda}\right) \\
& - \frac{1}{2} g_{\kappa\alpha} \partial_\nu g^{\alpha\beta}\left(\partial_\mu g_{\beta\lambda} + \partial_\lambda g_{\beta\mu} - \partial_\beta g_{\mu\lambda}\right) \\
& + g_{\kappa\alpha}\left(\Gamma^\alpha_{\mu\rho} \Gamma^\rho_{\nu\lambda} - \Gamma^\alpha_{\nu\rho} \Gamma^\rho_{\mu\lambda}\right).
\end{aligned}
\right] ②
$$

②を変形する。$\partial_\mu g^{\alpha\beta}$ 等について、行列 A の逆行列の微分の公式

$\partial_\mu A^{-1} = -A^{-1}(\partial_\mu A)A^{-1}$ より $g_{\kappa\alpha}\partial_\mu g^{\alpha\beta} = -\partial_\mu g_{\kappa\alpha}\cdot g^{\alpha\beta}$ 等が得られるから、

$$
\begin{aligned}
② = {} & -\frac{1}{2}\partial_\mu g_{\kappa\alpha}\cdot g^{\alpha\beta}\left(\partial_\nu g_{\beta\lambda} + \partial_\lambda g_{\beta\nu} - \partial_\beta g_{\nu\lambda}\right) \\
& + \frac{1}{2}\partial_\nu g_{\kappa\alpha}\cdot g^{\alpha\beta}\left(\partial_\mu g_{\beta\lambda} + \partial_\lambda g_{\beta\mu} - \partial_\beta g_{\mu\lambda}\right) + g_{\kappa\alpha}\left(\Gamma^\alpha_{\mu\rho}\Gamma^\rho_{\nu\lambda} - \Gamma^\alpha_{\nu\rho}\Gamma^\rho_{\mu\lambda}\right) \\
= {} & -\partial_\mu g_{\kappa\alpha}\Gamma^\alpha_{\nu\lambda} + \partial_\nu g_{\kappa\alpha}\Gamma^\alpha_{\mu\lambda} \\
& + \underbrace{g_{\kappa\alpha}\cdot\frac{1}{2}g^{\alpha\beta}}_{\delta^\beta_\kappa}\left(\partial_\mu g_{\beta\rho} + \partial_\rho g_{\beta\mu} - \partial_\beta g_{\mu\rho}\right)\Gamma^\rho_{\nu\lambda} \\
& - \underbrace{g_{\kappa\alpha}\cdot\frac{1}{2}g^{\alpha\beta}}_{\delta^\beta_\kappa}\left(\partial_\nu g_{\beta\rho} + \partial_\rho g_{\beta\nu} - \partial_\beta g_{\nu\rho}\right)\Gamma^\rho_{\mu\lambda} \\
= {} & -\partial_\mu g_{\kappa\alpha}\Gamma^\alpha_{\nu\lambda} + \partial_\nu g_{\kappa\alpha}\Gamma^\alpha_{\mu\lambda} \\
& + \frac{1}{2}\Gamma^\rho_{\nu\lambda}\left(\partial_\mu g_{\kappa\rho} + \partial_\rho g_{\kappa\mu} - \partial_\kappa g_{\mu\rho}\right) - \frac{1}{2}\Gamma^\rho_{\mu\lambda}\left(\partial_\nu g_{\kappa\rho} + \partial_\rho g_{\kappa\nu} - \partial_\kappa g_{\nu\rho}\right) \\
= {} & \frac{1}{2}\Gamma^\rho_{\nu\lambda}\left(-\partial_\mu g_{\kappa\rho} + \partial_\rho g_{\kappa\mu} - \partial_\kappa g_{\mu\rho}\right) - \frac{1}{2}\Gamma^\rho_{\mu\lambda}\left(-\partial_\nu g_{\kappa\rho} + \partial_\rho g_{\kappa\nu} - \partial_\kappa g_{\nu\rho}\right) \\
= {} & -g_{\rho\sigma}\Gamma^\sigma_{\kappa\mu}\Gamma^\rho_{\lambda\nu} + g_{\rho\sigma}\Gamma^\sigma_{\kappa\nu}\Gamma^\rho_{\lambda\mu}
\end{aligned}
$$

となる。ゆえに、①を得る。

（証明終わり）

スカラー曲率 R に対し、次の形の式が成り立つ：

$$\sqrt{-g}\,R = \sqrt{-g}\,G + \partial_\mu D^\mu, \qquad G: \partial_\lambda g_{\mu\nu} \text{ の2次式}.$$

(証明) 曲率テンソルは

$$R^\mu{}_{\lambda\alpha\beta} = \partial_\alpha \Gamma^\mu_{\beta\lambda} - \partial_\beta \Gamma^\mu_{\alpha\lambda} + \Gamma^\mu_{\alpha\rho}\Gamma^\rho_{\beta\lambda} - \Gamma^\mu_{\beta\rho}\Gamma^\rho_{\alpha\lambda}$$

で与えられるから、

Ricci テンソル $R_{\mu\nu} := R^\lambda{}_{\mu\lambda\nu} = \partial_\lambda \Gamma^\lambda_{\nu\mu} - \partial_\nu \Gamma^\lambda_{\lambda\mu} + \Gamma^\lambda_{\lambda\rho}\Gamma^\rho_{\nu\mu} - \Gamma^\lambda_{\nu\rho}\Gamma^\rho_{\lambda\mu}$,

$$R := g^{\mu\nu}R_{\mu\nu} = g^{\mu\nu}(\partial_\rho \Gamma^\rho_{\mu\nu} - \partial_\nu \Gamma^\rho_{\mu\rho} + \Gamma^\rho_{\gamma\rho}\Gamma^\gamma_{\mu\nu} - \Gamma^\rho_{\gamma\nu}\Gamma^\gamma_{\mu\rho})$$

となる。よって、

$$\begin{aligned}
\sqrt{-g}\,R &= \sqrt{-g}(g^{\mu\nu}\partial_\rho \Gamma^\rho_{\mu\nu} - g^{\mu\rho}\partial_\rho \Gamma^\nu_{\mu\nu}) - \sqrt{-g}\,g^{\mu\nu}(\Gamma^\rho_{\gamma\nu}\Gamma^\gamma_{\mu\rho} - \Gamma^\rho_{\gamma\rho}\Gamma^\gamma_{\mu\nu}) \\
&= \partial_\rho[\sqrt{-g}\,g^{\mu\nu}\Gamma^\rho_{\mu\nu} - \sqrt{-g}\,g^{\mu\rho}\Gamma^\nu_{\mu\nu}] - \partial_\rho(\sqrt{-g}\,g^{\mu\nu})\Gamma^\rho_{\mu\nu} + \partial_\rho(\sqrt{-g}\,g^{\mu\rho})\Gamma^\nu_{\mu\nu} \\
&\quad - \sqrt{-g}\,g^{\mu\nu}(\Gamma^\rho_{\gamma\nu}\Gamma^\gamma_{\mu\rho} - \Gamma^\rho_{\gamma\rho}\Gamma^\gamma_{\mu\nu}) \\
&= \partial_\rho D^\rho \underbrace{- \partial_\rho(\sqrt{-g}\,g^{\mu\nu})\Gamma^\rho_{\mu\nu}}_{①} \underbrace{+ \partial_\rho(\sqrt{-g}\,g^{\mu\rho})\Gamma^\nu_{\mu\nu}}_{②} - \sqrt{-g}\,\mathcal{G},
\end{aligned}$$

$$D^\rho := \sqrt{-g}\,g^{\mu\nu}\Gamma^\rho_{\mu\nu} - \sqrt{-g}\,g^{\mu\rho}\Gamma^\nu_{\mu\nu},$$

$$\mathcal{G} := g^{\mu\nu}(\Gamma^\rho_{\gamma\nu}\Gamma^\gamma_{\mu\rho} - \Gamma^\rho_{\gamma\rho}\Gamma^\gamma_{\mu\nu})$$

となる。① を計算する。まず、次の式が成り立つ：

$$\partial_\lambda \sqrt{-g} = \sqrt{-g}\,\Gamma^\mu_{\lambda\mu}.$$

(∵ $\Gamma^\kappa_{\lambda\mu} = \frac{1}{2}g^{\kappa\rho}(\partial_\lambda g_{\rho\mu} + \partial_\mu g_{\rho\lambda} - \partial_\rho g_{\lambda\mu})$ より $\Gamma^\mu_{\lambda\mu} = \frac{1}{2}g^{\mu\nu}\partial_\lambda g_{\mu\nu}$.

一方、

$$\partial_\lambda g = g\,g^{\mu\nu}\partial_\lambda g_{\mu\nu} \qquad ③$$

が成り立つから（後述）、

$$\Gamma^\mu_{\lambda\mu} = \frac{1}{2}\frac{1}{g}\partial_\lambda g = \frac{1}{\sqrt{-g}}\partial_\lambda\sqrt{-g}. \qquad \boxtimes \;)$$

これより

$$\begin{aligned}
① &= -\partial_\rho\sqrt{-g}\cdot g^{\mu\nu}\Gamma^\rho_{\mu\nu} - \sqrt{-g}\,\partial_\rho g^{\mu\nu}\Gamma^\rho_{\mu\nu} \\
&= -g^{\mu\nu}\sqrt{-g}\,\Gamma^\alpha_{\rho\alpha}\Gamma^\rho_{\mu\nu} - \sqrt{-g}\,\partial_\rho g^{\mu\nu}\Gamma^\rho_{\mu\nu} \\
&= -\sqrt{-g}\,(g^{\mu\nu}\Gamma^\alpha_{\rho\alpha} + \partial_\rho g^{\mu\nu})\Gamma^\rho_{\mu\nu}
\end{aligned}$$

となる。さらに次が成り立つ：

$$0 = \nabla_\rho g^{\mu\nu} = \partial_\rho g^{\mu\nu} + \Gamma^\mu_{\rho\alpha}g^{\alpha\nu} + \Gamma^\nu_{\rho\alpha}g^{\mu\alpha}.$$

(∵ 共変微分は Leibniz 則 $\partial_\rho(A_{\mu\nu}B^{\mu\nu}) = (\nabla_\rho A_{\mu\nu})B^{\mu\nu} + A_{\mu\nu}(\nabla_\rho B^{\mu\nu})$ を満たすことと、$\nabla_\rho g_{\mu\nu} = 0$ による。 $\boxtimes$)

したがって，①は次のようになる．

$$
\begin{aligned}
① &= \sqrt{-g}\left(-g^{\mu\nu}\Gamma^{\alpha}_{\rho\alpha} + \Gamma^{\mu}_{\alpha\rho}g^{\alpha\nu} + \Gamma^{\nu}_{\alpha\rho}g^{\mu\alpha}\right)\Gamma^{\rho}_{\mu\nu} \\
&= \sqrt{-g}\,g^{\mu\nu}\left(-\Gamma^{\alpha}_{\rho\alpha}\Gamma^{\rho}_{\mu\nu} + \Gamma^{\alpha}_{\mu\rho}\Gamma^{\rho}_{\alpha\nu} + \Gamma^{\alpha}_{\nu\rho}\Gamma^{\rho}_{\mu\alpha}\right) \\
&= \sqrt{-g}\,g^{\mu\nu}\left(-\Gamma^{\alpha}_{\rho\alpha}\Gamma^{\rho}_{\mu\nu} + 2\Gamma^{\alpha}_{\mu\rho}\Gamma^{\rho}_{\alpha\nu}\right).
\end{aligned}
$$

同様にして②は次のように計算される：

$$
② = -\sqrt{-g}\,g^{\mu\nu}\Gamma^{\rho}_{\mu\nu}\Gamma^{\gamma}_{\rho\gamma}.
$$

よって，

$$
①+② = 2\sqrt{-g}\,g^{\mu\nu}\left(-\Gamma^{\rho}_{\mu\nu}\Gamma^{\gamma}_{\rho\gamma} + \Gamma^{\alpha}_{\mu\rho}\Gamma^{\rho}_{\alpha\nu}\right) = 2\sqrt{-g}\,G
$$

となる．

以上より，次を得る：

$$
\begin{aligned}
&\sqrt{-g}\,R = \sqrt{-g}\,G + \partial_\rho D^\rho, \\
&G = g^{\mu\nu}\left(\Gamma^{\rho}_{\gamma\nu}\Gamma^{\gamma}_{\mu\rho} - \Gamma^{\rho}_{\gamma\rho}\Gamma^{\gamma}_{\mu\nu}\right) \qquad (\partial_\lambda g_{\mu\nu} \text{ の2次式}), \\
&D^\mu = \sqrt{-g}\left(g^{\rho\nu}\Gamma^{\mu}_{\rho\nu} - g^{\rho\mu}\Gamma^{\nu}_{\rho\nu}\right).
\end{aligned}
$$

③の証明が残っている．これは，一般に$N\times N$正則行列 $A=(a_{kl})$ の行列式の変分 $\delta|A|$ が次のように表されることから言える：

$$
\delta|A| = |A|\sum_{k,l=1}^{N}(A^{-1})_{lk}\,\delta a_{kl} \qquad \left((A^{-1})_{lk} : A^{-1}\text{の}(l,k)\text{成分}\right). \quad ―④
$$

実際，

$$
\delta|A| = \begin{vmatrix} \delta a_{11} & \cdots & \delta a_{1N} \\ a_{21} & \cdots & a_{2N} \\ \vdots & & \vdots \\ a_{N1} & \cdots & a_{NN} \end{vmatrix} + \begin{vmatrix} a_{11} & \cdots & a_{1N} \\ \delta a_{21} & \cdots & \delta a_{2N} \\ \vdots & & \vdots \\ a_{N1} & \cdots & a_{NN} \end{vmatrix} + \cdots + \begin{vmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & & \vdots \\ \delta a_{N1} & \cdots & \delta a_{NN} \end{vmatrix}
$$

$$
\begin{aligned}
&= \sum_{k,l=1}^{N}(-1)^{k+l}\,\delta a_{kl} \times \underbrace{\det\begin{bmatrix}A\text{から第}k\text{行第}l\text{列を} \\ \text{除いた小行列}\end{bmatrix}}_{\tilde{a}_{kl}\text{ とおく}} \\
&= \sum_{k,l=1}^{N}(-1)^{k+l}\,\tilde{a}_{kl}\,\delta a_{kl}
\end{aligned}
$$

が成り立ち，逆行列 A^{-1} の (k,l) 成分が $(A^{-1})_{kl} = \dfrac{1}{|A|}(-1)^{k+l}\tilde{a}_{lk}$ と表されることから，④を得る．

（証明終わり）

Schwarzschild 計量の導出

計算しやすいように計量成分を指数関数を用いて表す.

$$g_{00} = -e^{-\nu(r)},\quad g_{rr} = e^{\lambda(r)},\quad g_{\theta\theta} = r^2,\quad g_{\varphi\varphi} = r^2 \sin^2\theta,$$

$$g^{00} = -e^{\nu(r)},\quad g^{rr} = e^{-\lambda(r)},\quad g^{\theta\theta} = r^{-2},\quad g^{\varphi\varphi} = r^{-2}\sin^{-2}\theta.$$

$$\Gamma^{\kappa}_{\mu\nu} = \frac{1}{2} g^{\kappa\lambda}(\partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu})$$

に代入して次を得る.

$$\Gamma^{0}_{r0} = \Gamma^{0}_{0r} = \frac{1}{2}\nu'(r),$$

$$\Gamma^{r}_{00} = \frac{1}{2}\nu'(r)e^{-\lambda(r)+\nu(r)},\quad \Gamma^{r}_{rr} = \frac{1}{2}\lambda'(r),\quad \Gamma^{r}_{\theta\theta} = -re^{-\lambda(r)},$$

$$\Gamma^{r}_{\varphi\varphi} = -re^{-\lambda(r)}\sin^2\theta$$

$$\Gamma^{\theta}_{\varphi\varphi} = -\sin\theta\cos\theta,\quad \Gamma^{\theta}_{r\theta} = \Gamma^{\theta}_{\theta r} = \Gamma^{\varphi}_{r\varphi} = \Gamma^{\varphi}_{\varphi r} = \frac{1}{r},$$

$$\Gamma^{\varphi}_{\theta\varphi} = \Gamma^{\varphi}_{\varphi\theta} = \cot\theta,$$

その他の Γ の成分 $= 0$.

これらを用いて次の微分方程式を得る.

$$R_{00} = 0 \rightarrow \frac{1}{2}\nu'' + \frac{1}{4}\nu'^2 - \frac{1}{4}\nu'\lambda' + \frac{\nu'}{r} = 0. \quad \text{—①}$$

$$R_{rr} = 0 \rightarrow -\frac{1}{2}\nu'' + \frac{1}{4}\nu'\lambda' + \frac{\lambda'}{r} - \frac{\nu'^2}{4} = 0. \quad \text{—②}$$

$$R_{\theta\theta} = 0 \rightarrow 1 + e^{-\lambda}\left(\frac{1}{2}r\lambda' - \frac{1}{2}r\nu' - 1\right) = 0, \quad \text{—③}$$

$$R_{\varphi\varphi} = 0 \rightarrow 1 + e^{-\lambda}\left(-1 + \frac{1}{2}r\lambda' - \frac{1}{2}r\nu'\right) = 0. \quad \text{—④}$$

* その他の $R_{\mu\nu}$ ははじめから 0 である.

①+② より $\lambda' + \nu' = 0$, $\nu + \lambda = c$ (const.).

これを③に代入して,

$$e^{-\lambda}(1 - r\lambda') = 1,\quad (re^{-\lambda})' = 1,\quad re^{-\lambda} = r - r_s \quad (r_s : \text{const.}).$$

$$\therefore\quad e^{\lambda} = \frac{r}{r - r_s} = \left(1 - \frac{r_s}{r}\right)^{-1},$$

$$e^{\nu} = e^{c-\lambda} = e^{c}\left(1 - \frac{r_s}{r}\right).$$

これらの解 $\lambda(r)$, $\nu(r)$ は③をみたす.

$$ds^2 = -e^{c}\left(1 - \frac{r_s}{r}\right)(dx^0)^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

とくに $e^c = 1$ とおいたものが Schwarzschild 計量である. ⊠